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# ANTIPHASE SYNCHRONIZATION IN SYMMETRICALLY COUPLED SELF-OSCILLATORS

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We consider the antiphase synchronization in symmetrically coupled self-oscillators. As model, two Chua's circuits coupled via a capacity are used. Linear analysis in the vicinity of the symmetric subspace gives the stability conditions for antiphase oscillations. Numerical oscillations demonstrate controlled antiphase synchronization at different values of the parameters of the system.

## 1. Introduction

Since the beginning of the 90s a new direction appeared in the theory of dynamical systems, which was called "Controlling chaos". The subject of this field is how to transform chaotic behavior to a regular one or to a more simple chaotic one by means of special small influence on the dynamical system. Pioneer works of this new direction were papers by Hubler and Lucher [1989], Jackson [1990] and the well-known paper by Ott, Grebogi and Yorke [1990]. Now there is a lot of different algorithms of chaos control, which are applied to tasks of hydrodynamics [Singer *et al.*, 1991], mechanics [In & Ditto, 1995; Baretto & Grebogi, 1995], chemistry [Petrov *et al.*, 1993], biology and medicine [Garfinkel *et al.*, 1992; Schiff *et al.*, 1994].

Methods of control of chaos can be applied for synchronization of coupled chaotic oscillators. In scientific literature there is no common view for the problem of chaotic synchronization. Different types of chaotic synchronization are "complete synchronization", when oscillations of subsystems are equal or nearly equal to each other [Fujisaka & Yamada, 1983; Afraimovich *et al.*, 1986]; "generalized synchronization", when there is a functional dependence between states of the subsystems [Rulkov *et al.*, 1995]; "frequency synchronization" which means locking of peaks in the spectra of oscillations [Anishchenko *et al.*, 1991, 1992]; "phase synchronization" when phases of oscillations are locked while amplitudes remain uncorrelated [Rosenblum *et al.*, 1996]. The majority of works on controlled synchronization of chaos consider the case of complete in-phase synchronization [Lai & Grebogi, 1993; Malescio, 1996]. Another interesting case of chaotic synchronization, which can be realized by chaos control is antiphase chaotic synchronization, when states of the subsystems satisfy the condition $\mathbf{x}_1 = -\mathbf{x}_2$. The antiphase synchronization of chaos on the example of a two-dimensional map and a six-dimensional flaw was considered in the work [Cao & Lai, 1998]. Following Pecora and Carroll [1990] they used the so-called "master–slave" approach to the task of the antiphase synchronization. In our work we consider antiphase synchronization in symmetrically coupled identical self-oscillators with additional controlling feedback loop.

Let us consider the task of antiphase synchronization in symmetrically coupled oscillators. Let the equation of the system be in the form:

$$\begin{aligned}\dot{\mathbf{x}}_1 &= \mathbf{f}(\mathbf{x}_1) + \gamma(\mathbf{x}_1, \mathbf{x}_2)\\ \dot{\mathbf{x}}_2 &= \mathbf{f}(\mathbf{x}_2) + \gamma(\mathbf{x}_2, \mathbf{x}_1)\end{aligned} \qquad (1)$$

$$\gamma(\mathbf{x}_1, \mathbf{x}_2) = -\gamma(\mathbf{x}_2, \mathbf{x}_1)\,,$$

$\mathbf{x}_{1,2} \in R^N$ are vectors of dynamical variables,
$\mathbf{f} \in R^N$ — vector function which determines the right side of the equation of the single oscillator,
$\gamma \in R^N$ — vector function, which determines the symmetric coupling between oscillators.

The possibility of the antiphase synchronization in the system (1) is determined by two factors. Firstly, this is an existence of the symmetric subspace $\mathbf{x}_1 = -\mathbf{x}_2$ in the phase space of the system, secondly, this is stability of antiphase motions to small perturbations in the normal direction to the subspace. The first condition is realized when Eq. (1) is invariant to the transformation of variables:

$$\mathbf{x}_1 \leftrightarrow -\mathbf{x}_2 \,. \tag{2}$$

Substituting (2) into (1) we get the conditions for the functions $\mathbf{f}(\mathbf{x})$ and $\gamma(\mathbf{x}_1, \mathbf{x}_2)$:

$$\begin{aligned} \mathbf{f}(-\mathbf{x}) &= -\mathbf{f}(\mathbf{x}) \\ \gamma(-\mathbf{x}_1, -\mathbf{x}_2) &= -\gamma(\mathbf{x}_1, \mathbf{x}_2) \end{aligned} \tag{3}$$

To determine the stability conditions for antiphase oscillations it is convenient to use a transformation of variables:

$$\begin{aligned} \mathbf{u} &= \frac{\mathbf{x}_1 + \mathbf{x}_2}{2}\,, \\ \mathbf{v} &= \frac{\mathbf{x}_1 - \mathbf{x}_2}{2} \end{aligned} \tag{4}$$

in the vicinity of the symmetric subspace $\mathbf{x}_1 = -\mathbf{x}_2$. The equations in new variables near the subspace can be written in the form:

$$\dot{\mathbf{u}} = \mathbf{f}'(\mathbf{v})\mathbf{u} \tag{5}$$

$$\dot{\mathbf{v}} = \mathbf{f}(\mathbf{v}) + \gamma(\mathbf{v}, -\mathbf{v})\,. \tag{6}$$

Equation (6) describes the dynamics of the system inside the symmetric subspace. It determines the equation of the equivalent single oscillator. Unlike the case of the in-phase synchronization the parameters of the equivalent oscillator depend on the coupling vector $\gamma$.

Equation (5) describes the dynamics of the system (1), (3) in the vicinity of the symmetric subspace in the direction normal to it. A zero solution of (5) $\mathbf{u} = 0$ corresponds to antiphase motions in the original system. Stability of this solution means stability of the synchronous oscillations.

For controlling the stability of the antiphase motions, let us add a feedback term to the right side of Eq. (1):

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{f}(\mathbf{x}_1) + \gamma(\mathbf{x}_1, \mathbf{x}_2) + \Phi_1(\mathbf{x}_1, \mathbf{x}_2) \\ \dot{\mathbf{x}}_2 &= \mathbf{f}(\mathbf{x}_2) + \gamma(\mathbf{x}_2, \mathbf{x}_1) + \Phi_2(\mathbf{x}_1, \mathbf{x}_2)\,. \end{aligned} \tag{7}$$

The controlling influence will not induce new antiphase oscillating regimes in the system, but will only change the stability of the old ones, if the controlling function $[\Phi_1 \ \Phi_2]^T$ is equal to zero on the symmetric subspace and is not equal to zero on the subspace normal to it. This leads to the condition:

$$\Phi_{1,2}(\mathbf{x}, -\mathbf{x}) = 0\,, \quad \Phi_{1,2}(\mathbf{x}, \mathbf{x}) \neq 0\,. \tag{8}$$

It is convenient to choose the controlling function in linear form on both arguments:

$$\Phi_{1,2} = [r_{1,2}](\mathbf{x}_1 + \mathbf{x}_2)\,, \tag{9}$$

$$[r_{1,2}] \in R^N \times R^N\,.$$

For new variables $\mathbf{u}$ and $\mathbf{v}$ the equations with control are the following:

$$\dot{\mathbf{u}} = [\mathbf{f}'(\mathbf{v}) + [r_1] + [r_2]]\mathbf{u} \tag{10}$$

$$\dot{\mathbf{v}} = \mathbf{f}(\mathbf{v}) + \gamma(\mathbf{v}, -\mathbf{v}) + ([r_1] - [r_2])\mathbf{u} \tag{11}$$

Unknown matrixes $[r_1]$ and $[r_2]$ are determined from linear analysis of the stability of the fixed point $\mathbf{u} = 0$ of Eq. (10).

## 2. The System Under Consideration

As an object for investigations we used the system of two coupled via a capacity identical Chua's oscillators. The scheme of the circuit is presented in Fig. 1(a), and the characteristic of the nonlinear element is in Fig. 1(b). The dynamics of the single Chua's generator is well described in scientific literature (see e.g. [Komuro *et al.*, 1991]). It demonstrates transition from order to chaos via a cascade of subharmonic bifurcations and then the uniting of chaotic attractors symmetric to each other by forming the so-called "double-scroll" attractor. The equation of the coupled oscillators in dimensionless

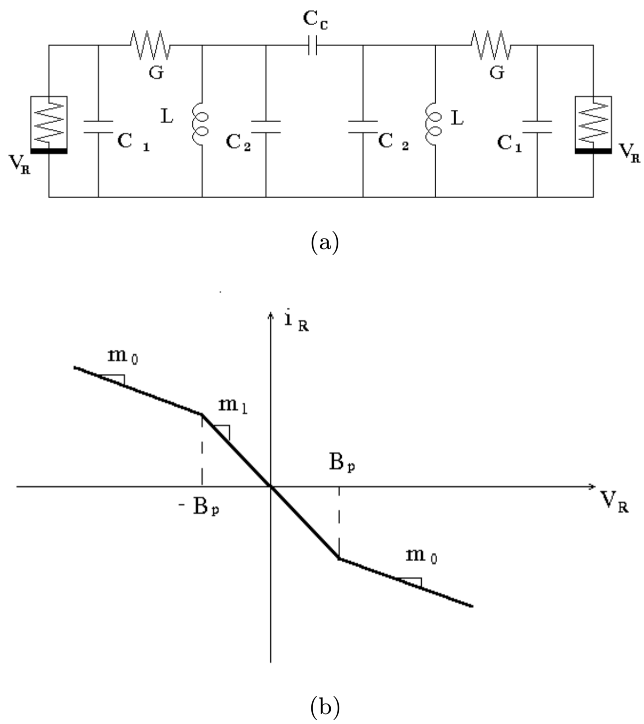



Fig. 1. (a) The scheme of the two coupled via a capacity Chua's oscillators and (b) the volt–ampere characteristic of the nonlinear element.

form can be written as follows:

$$\begin{aligned} \dot{x}_1 &= \alpha(y_1 - x_1 - f(x_1)) \\ \dot{y}_1 &= x_1 - y_1 + z_1 + \gamma((x_2 - x_1) \\ &\quad - (y_2 - y_1) + (z_2 - z_1)) \\ \dot{z}_1 &= -\beta y_1 \\ \dot{x}_2 &= \alpha(y_2 - x_2 - f(x_2)) \\ \dot{y}_2 &= x_2 - y_2 + z_2 + \gamma((x_1 - x_2) \\ &\quad - (y_1 - y_2) + (z_1 - z_2)) \\ \dot{z}_2 &= -\beta y_2 \,, \end{aligned} \tag{12}$$

$$f(x) = \begin{cases} bx + a - b & \text{if } x \geq 1 \\ ax & \text{if } |x| < 1 \\ bx - a + b & \text{if } x \leq -1 \end{cases}$$

where $\alpha = C_2/C_1$, $\beta = C_2/(LG^2)$, $\gamma = C_c/(C_2 + 2C_c)$, $a = m_1/G$, $b = m_0/G$, $\tau = tG/C_2$. The parameters $\alpha$ and $\gamma$ were chosen as the varied ones, the parameters $\beta$, $a$, $b$ were fixed to values: $\beta = 22$, $a = -8/7$, $b = -5/7$.

The system (12) has nine equilibria:

$$\begin{aligned} &\{P_1, P_1\}\,, \quad \{P_1, P_0\}\,, \quad \{P_1, P_2\}\,, \\ &\{P_0, P_1\}\,, \quad \{P_0, P_0\}\,, \quad \{P_0, P_2\}\,, \\ &\{P_2, P_1\}\,, \quad \{P_2, P_0\}\,, \quad \{P_2, P_2\}\,, \end{aligned}$$

where $P_1 = (1.5,\ 0,\ -1.5)$, $P_0 = (0,\ 0,\ 0)$ and $P_2 = (-1.5,\ 0,\ 1.5)$ are equilibria of the single Chua's oscillator.

The system (12) has two basic kinds of symmetry: $R : x_1 \leftrightarrow x_2,\ y_1 \leftrightarrow y_2,\ z_1 \leftrightarrow z_2$ and $I : x_{1,2} \leftrightarrow -x_{1,2},\ y_{1,2} \leftrightarrow -y_{1,2},\ z_{1,2} \leftrightarrow -z_{1,2}$. Because transformations $R$ and $I$ are commutative of each other: $I \circ R = R \circ I$, their combination is also the symmetry for the system (12): $I \circ R : x_1 \leftrightarrow -x_2$, $y_1 \leftrightarrow -y_2$, $z_1 \leftrightarrow -z_2$. The symmetry of the system to the transformation $R$ is a necessary condition for the in-phase synchronization. The symmetry to the transformation $I \circ R$ is a necessary condition for the antiphase synchronization.

The dynamics of the system (12) is described in [Astakhov *et al.*, 1997a]. It demonstrates transition to chaos through both period-doubling bifurcations and tori breaking. The system is characterized by multistability when several attractors coexist in the phase space. As in the cases of other symmetrically coupled oscillators with period-doublings (see e.g. [Astakhov *et al.*, 1989; Anishchenko *et al.*, 1995]) the origin of the multistability in the system is the bifurcational mechanisms, when every cycle in the cascade of the period-doubling undergoes this bifurcation twice. The first period-doubling occurs with the stable cycle, as a result the cycle loses its stability and in its neighborhood a stable cycle of double period appears. The second period-doubling occurs with the saddle cycle, as a result the cycle gets another direction of instability and in its neighborhood a saddle cycle of double period appears. This scenario leads to the increasing of a number of cycles with equal periods. There are two cycles with period-two, four with period-four, eight with period-eight, etc. Cycles with equal periods have different time-delays between oscillations of the subsystems. The value of the delays are proportional to the period of the original period-one cycle. On bases of these cycles different families of multiband chaotic attractors are formed. For further parameter changes, in the chaotic region there are bound-merging bifurcations which are accompanied by uniting attractors of different families. The scheme of development for multistability in the system is presented in Fig. 2. There the letter denotes the type of the limit set: C is a cycle, T is a two-dimensional torus, A is a chaotic set; the first index is a number of connected regions on the Poincare section, the upper one is a time-delay between oscillations of the subsystems per the period of the original period-one cycle (for periodic oscillations).

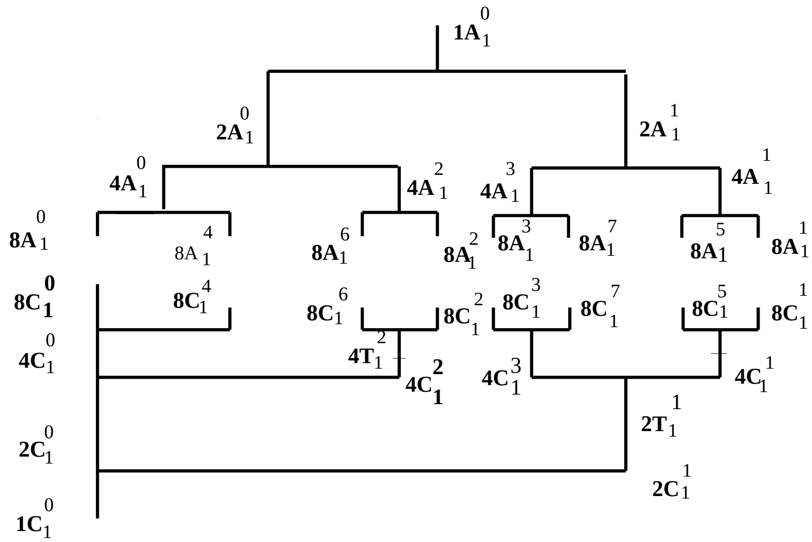



Fig. 2. The diagram of the appearance of different oscillating regimes in the system of coupled oscillators.

## 3. Control of Antiphase Synchronization in the System

In-phase and antiphase chaotic oscillations does not take place in the system (12) without control influence. To realize a regime of antiphase synchronization we add a term of the feedback loop $r(x_1 + x_2)$ to the equation of the system:

$$\dot{x}_1 = \alpha(y_1 - x_1 - f(x_1)) + r(x_1 + x_2) \tag{13}$$

$$\dot{y}_1 = x_1 - y_1 + z_1 + \gamma((x_2 - x_1) - (y_2 - y_1) + (z_2 - z_1)) \tag{14}$$

$$\dot{z}_1 = -\beta y_1 \tag{15}$$

$$\dot{x}_2 = \alpha(y_2 - x_2 - f(x_2)) \tag{16}$$

$$\dot{y}_2 = x_2 - y_2 + z_2 + \gamma((x_1 - x_2) - (y_1 - y_2) + (z_1 - z_2)) \tag{17}$$

$$\dot{z}_2 = -\beta y_2 \,. \tag{18}$$

Let us choose new variables in the form:

$$u = \frac{x_1 + x_2}{2}, \quad u' = \frac{x_1 - x_2}{2}$$
$$v = \frac{y_1 + y_2}{2}, \quad v' = \frac{y_1 - y_2}{2}$$
$$w = \frac{z_1 + z_2}{2}, \quad w' = \frac{z_1 - z_2}{2}.$$

We add and subtract Eqs. (13) and (16), (14) and (17), (15) and (18) respectively:

$$\begin{aligned}
\dot{u} &= \alpha(v - u - 0.5(f(x_1) + f(x_2)) + ru \\
\dot{v} &= u - v + w \\
\dot{w} &= -\beta v \\
\dot{u}' &= \alpha(v' - u' - 0.5(f(x_1) - f(x_2)) + ru \\
\dot{v}' &= (1 - 2\gamma)(u' - v' + w') \\
\dot{w}' &= -\beta v'
\end{aligned} \tag{19}$$

If the phase point locates near the symmetric subspace $\mathbf{x}_1 = -\mathbf{x}_2$ the following conditions will be fulfilled:

$$x_1 > 1 \quad \text{and} \quad x_2 < -1$$
$$x_1 < -1 \quad \text{and} \quad x_2 > 1 \,,$$

or

$$|x_1| < 1 \quad \text{and} \quad |x_2| < 1 \,.$$

Therefore equations of the oscillators with control in new variables have the form:

$$\dot{u} = \alpha(v - u - h(u')u) + ru \tag{20}$$

$$\dot{v} = u - v + w \tag{21}$$

$$\dot{w} = -\beta v \tag{22}$$

$$\dot{u}' = \alpha(v' - u' - f(u')) + ru \tag{23}$$

$$\dot{v}' = (1 - 2\gamma)(u' - v' + w') \tag{24}$$

$$\dot{w}' = -\beta v' \,, \tag{25}$$

where

$$h(u') = \begin{cases} b, & \text{if } |u'| > 1 \\ a, & \text{if } |u'| < 1, \end{cases}$$

Equations (23)–(25) describe the dynamics of the system inside the symmetric subspace. Without control, if $r = 0$ they can be written in the form:

$$\begin{aligned} \frac{1}{1-2\gamma}\dot{u}' &= \alpha'(v' - u' - f(u')) \\ \frac{1}{1-2\gamma}\dot{v}' &= u' - v' + w' \\ \frac{1}{1-2\gamma}\dot{w}' &= -\beta' v', \end{aligned} \tag{26}$$

where

$$\alpha' = \frac{\alpha}{1-2\gamma}, \quad \beta' = \frac{\beta}{1-2\gamma}.$$

Or,

$$\begin{aligned} \frac{du'}{d\tau'} &= \alpha'(v' - u' - f(u')) \\ \frac{dv'}{d\tau'} &= u' - v' + w' \\ \frac{dw'}{d\tau'} &= -\beta' v', \end{aligned} \tag{27}$$

here $\tau' = (1-2\gamma)\tau$ is "slow" time.

It is seen that the form of oscillations in the symmetric subspace is the same as in the single oscillator with other parameters $\alpha$ and $\beta$:

$$\alpha \to \frac{\alpha}{1-2\gamma}$$
$$\beta \to \frac{\beta}{1-2\gamma},$$

and all time scales are changed $(1-2\gamma)$ times.

The stability of antiphase oscillations is determined by the stability of the fixed point ($u = 0$, $v = 0$, $w = 0$) [Eqs. (20)–(22)]. Antiphase oscillations will be surely stable if this point is stable at any point ($u'$, $v'$, $w'$) on the attractor. To determine the values of the coefficient $r$ that give sufficient condition for stabilization of the antiphase motions we hold linear analysis of the fixed point. The Jacobi matrix for the system is:

$$[J] = \begin{bmatrix} -\alpha c + r & \alpha & 0 \\ -1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$$

where

$$c = \begin{cases} 1+b & \text{if } |u'| > 1 \\ 1+a & \text{if } |u'| < 1 \end{cases}$$

The characteristic equation for the matrix eigenvalues has the form:

$$\lambda^3 + \lambda^2[\alpha c - r + 1] + \lambda[\alpha c - r + \beta - \alpha] + \beta(\alpha c - r) = 0 \tag{28}$$

Using the Rauth–Gurvitz criterion gives the following condition for the coefficient $r$:

$$r < \alpha c - \frac{\alpha - 1}{2} - \frac{\sqrt{(\alpha-1)^2 - 4(\beta - \alpha)}}{2}. \tag{29}$$

For evaluating $r$ we choose the minimal possible value of $c$:

$$c = \min(1+a, 1+b) = 1+a.$$

The region of correspondent values of $r$ is located under the line in Fig. 3. This condition is sufficient for antiphase synchronization. It leads local attracting trajectories to the symmetric subspace at any of its points. This is a stronger condition than that really needed for synchronization, but this can ensures it.

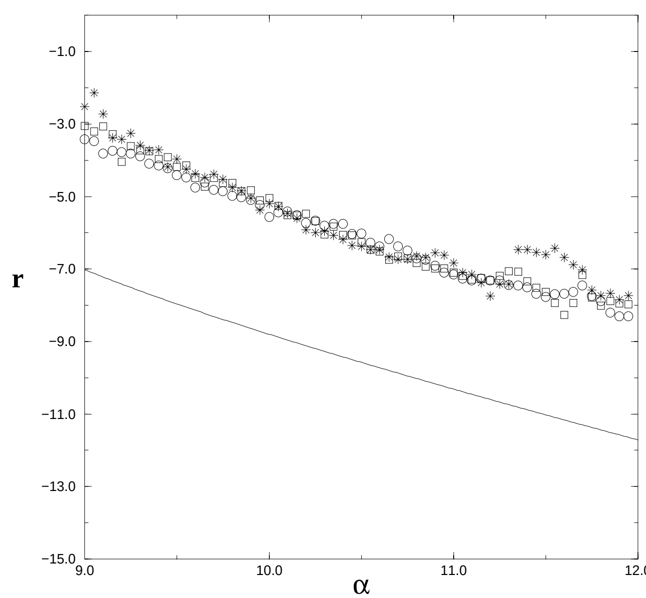



Fig. 3. The region on the parameters plane ($\alpha - r$) where antiphase oscillations are stable. The solid line is obtained from the linear analysis, points are obtained from computer experiments: (○) at $\gamma = 0.05$, (□) at $\gamma = 0.1$ and (*) at $\gamma = 0.15$.

## 4. Experimental Investigations of Controlled Antiphase Synchronization

At determined parameter values in the system (12) there is double–double scroll chaotic attractor [Figs. 4(a) and 4(b)] which includes saddle sets located in the symmetric subspace $\mathbf{x}_1 = -\mathbf{x}_2$. Because the chaotic trajectory visits every point of the attractor, it enters a small neighborhood of the symmetric subspace. The measure of the nearness of the current phase point to the subspace can be chosen in the form:

$$\rho = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2 + (z_1 + z_2)^2}\,.$$

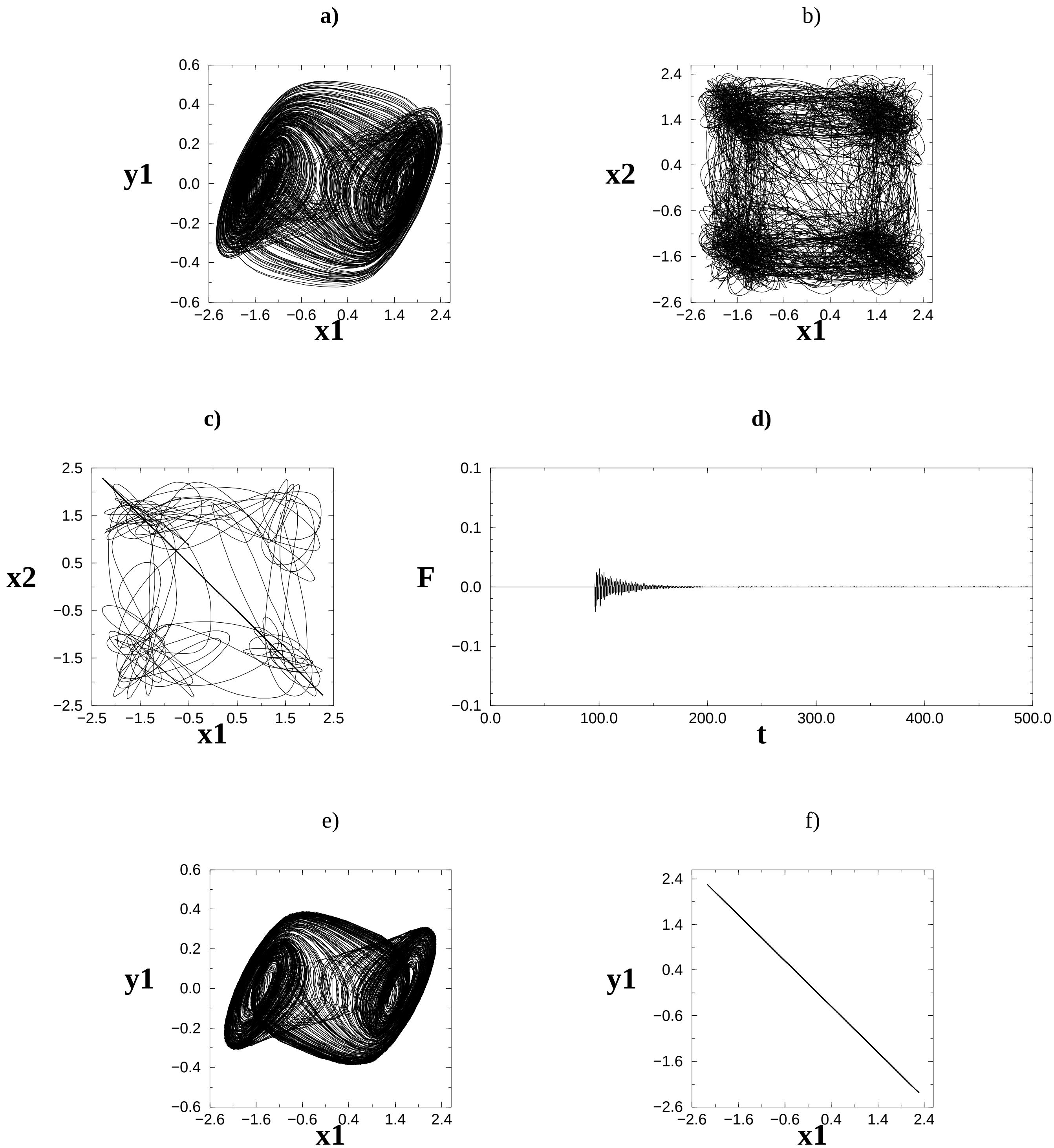
Let us choose a small value $\varepsilon > 0$ as a criterion for the nearness of the phase point to the symmetric subspace.

For numeric experiments of the investigation of controlled antiphase synchronization we used the following algorithm. For the current value $\alpha$ we found the value of the parameter $r$ which satisfies the condition (29). The system (13)–(18) begins to oscillate from any initial conditions in the basins of the chaotic attractor which includes saddle antiphase motions. During oscillations we continuously seek for the value $\rho$ which is compared with the chosen value $\varepsilon$. While $\rho > \varepsilon$ the controlling influence is switched off and $r = 0$. When the phase point appears in the $\varepsilon$-neighborhood of the symmetric subspace, the controlling influence begins operating. The antiphase motions become stable to transverse to the symmetric subspace perturbations and the trajectory is attracted to the subspace. The system transits to the regime of antiphase synchronization and stays there while the control influence is switched on. In Fig. 4 there is an example of the controlled transition from the double–double scroll regime to antiphase chaotic oscillations. Figures 4(a) and 4(b) present projections of the phase portrait of the chaotic attractor in the system without control. Figures 4(c) and 4(d) demonstrate the process of the controlled transition from the nonsynchronous oscillations to synchronous ones. In Fig. 4(d) there is dependence of the controlling term $F = r(x_1 + x_2)$ on time $t$. It is seen that the control influence becomes very small when the aim of the control is achieved. In Figs. 4(e) and 4(f) there are projections of the phase portraits of the resulting synchronous chaotic attractor.

We determined the dependence of the minimal value of the parameter $r$ on $\alpha$ at different values $\gamma$ which allow the synchronization of the oscillators. Initial conditions were chosen near the symmetric subspace. The results are presented in Fig. 3. Regions of stable antiphase oscillations are located under the boundaries which are marked by symbols ($\circ$) for $\gamma = 0.05$, ($\square$) for $\gamma = 0.01$ and (*) for $\gamma = 0.15$. It is seen that the boundary does not practically depend on the coupling coefficient and its form is very similar to the theoretical curve (the solid line), though the experimental boundaries are located over the curve.

In our computer experiments we also considered the question how the system exits the regime of antiphase synchronization on reducing the absolute value of the controlling parameter $r$. We chose the parameters values: $\alpha = 10.4$ and $\gamma = 0.2$ that correspond to the one-band chaotic attractor located in the symmetric subspace and added noise of small intensity to the system. In these experiments we did not switch off control influence when the trajectory leaves the neighborhood of the subspace. In Fig. 5 there are consequence changing of the phase portrait projections and the time series $x_1(t) + x_2(t)$ at the parameter $r$ increasing from $r = -6.43$ till $r = -6.36$. At $r = -6.43$ [Fig. 5(a)] oscillations are antiphase. At $r \sim -6.406$ [Fig. 5(b)] a short burst of small values appear, but oscillations remain nearly antiphase. On further increasing $r$ these bursts appear more often. Then, at $r = -6.395$ we see intermittency process between the mentioned nearly synchronous oscillations and bursts of large "amplitude". These bursts have duration of $\sim 10^4$ and are interrupted by intervals of near synchronous oscillations [Fig. 5(c)]. The chaotic attractor changes its structure. The chaotic trajectory begins to visit neighborhoods of both equilibria $\{P_1, P_2\}$ and $\{P_2, P_1\}$. The projection of the phase portrait becomes more "thick" and oscillations become partially synchronous. On further increase of $r$ the "bursts" appear more often, their "amplitude" and duration increase. Oscillations become less and less synchronous [Fig. 5(d)]. Then, at $r \sim -6.35$ the chaotic set becomes unattractive and the trajectory leaves to infinity.

Considering the above phenomena these remain questions about bifurcational mechanisms which lead to antiphase synchronization loss. Are these mechanisms similar to the mechanisms of the in-phase synchronization loss [Astakhov *et al.*, 1997b]?




$\alpha = 12.3,\ \gamma = 0.2$

Fig. 4. (a and b) Projections of phase portraits and time series of oscillations without control of chaos, (c and d) transition process to antiphase oscillations and (e and f) resulting oscillations using chaos control.

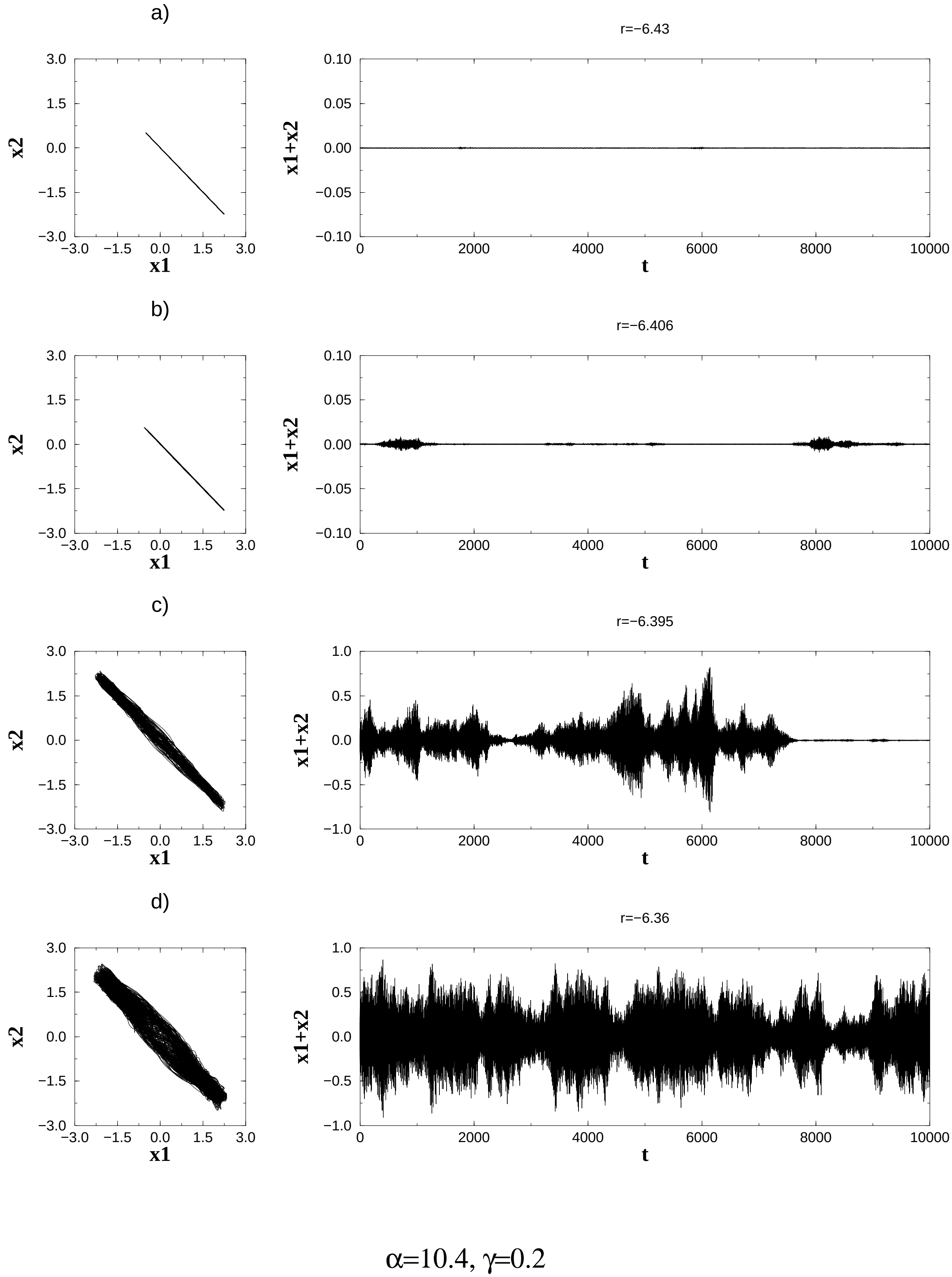



Fig. 5. Projections of phase portraits and time series of oscillations at consequent reduction of the absolute value of the controlling parameter $r$.

Do periodic cycles which form the "skeleton" of the attractor play important role in these mechanisms? The solution to this problem needs further investigations.

## 5. Conclusion

In this work, we considered the possibility of controlled antiphase synchronization in the system of two symmetrically coupled Chua's oscillators by means of additional feedback loop. By the linear analysis of the stability of antiphase oscillations to transversal perturbations we found sufficient conditions for the stability of antiphase regimes. We also showed that the system of two coupled oscillators in this regime behaves as the single Chua's oscillator with transformed parameters and with "slow" time. Numeric experiments confirmed that antiphase oscillations are stabilized in the chosen region of the controlling parameter.

## Acknowledgment

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